### Title:

TANAIF Theoretical and numerical aspects in fluid dynamics and turbulent flows

# Credit value:

3 ECTS

# Mandatory/Optional:

Optional

#### Semester:

2

## Lecturer/s:

Luis Vega (UPV/EHU) Francisco De La Hoz (UPV/EHU) Carlos Gorria (UPV/EHU)

### **University:**

University of the Basque Country

### **Department:**

Department of Mathematics

### **Rationale:**

This course is devoted to the modeling of equations of fluid dynamics in the presence of turbulence, vortices and stochastic flows. The knowledge about these dynamics is essential for the design of durable mechanisms that optimize the capture of energy from either wind or sea forces. Some nonlinear PDE which are susceptible of being studied analytically and numerically are relevant within this context. Moreover, the stochastic Burgers and Navier-Stokes equations are useful in the presence of nondeterministic forces.

### **Objectives:**

To provide students with a mathematical description of the emergence and propagation of some types of singularities in fluid dynamics. We show the equations modeling these phenomena and we analyze some particular solutions of special interest. Due to the limitations of analytical methods for solving nonlinear partial differential equations and stochastic differential equations that appear in this framework, some sophisticated numerical schemes are proposed. Students will focus on the programming of numerical methods to make an efficient use of them.

**<u>Skills:</u>** (according to the list of skills provided)

Subject skills		REM Master Skills					
	L2.1	L2.2	L2.3	L2.4	L2.5	L2.6	L2.7
L3.1. Learning about the partial differential			Х	Х			
equations of fluid dynamics, the physical laws that							
lead into these equations and the assumptions							
taken in the formulation.							
L3.2. Knowing the mathematical concepts of	Х					Х	Х
vortex filaments, sheets and patches and							
visualizing the evolution of these types of							
solutions.							
L3.3. Understanding the concept of stochastic		Х		Х			
forces in fluids and introducing its effect into the							
equations.							
L3.4. Being able to program nontrivial numerical	Х						Х
schemes to solve partial differential equations							
with singularities and stochastic differential							
equations.							

# **Teaching and learning methods:**

The subject will be taught principally by master lectures, besides there will be some practical sessions as well as computing practices.

# Allocation of student time:

	Attendance (classroom, lab,)	Non attendance (lecture preparation, self study)
Lectures	22 hours	
Computer Lab	8 hours	

### Assessment:

Home assignments (made individually) 50% and computer programs 50%.

# Assessment Matrix:

Subject	Assessment method				
skills	Home work	<b>Computer programs</b>	Report		
L3.1.	100%				
L3.2.		60%	40%		
L3.3.	100%				
L3.4.		60%	40%		

## Programme:

	-
Lesson 1	Basic notions of turbulent flows
	From the Euler equations and by taking some assumptions, the formulation can be
	transformed into some nonlinear PDE suitable for being studied analytically and numerically.
	Distribution (8 h theory + 2 h practical classroom)
-	Distribution (on mony + 2 n product classroom)
Lesson 2	Vortex filaments, sheets and patches
	The analysis of the self-similar solutions of the PDE involved in fluid dynamics and the
	numerical simulations help to understand the origin and evolution of singularities
	numerieur simulations help to understand the origin and evolution of singularities.
	Distribution (6 h theory + 4 h computer)
Lesson 3	Burgers and Stokes equations under stochastic forces
	In the presence of nondeterministic forces the model turns to the stochastic Burgers and
	Navier-Stokes equations. Design of implicit numerical methods for approximate solutions.
	Distribution (6 h theory $+ 4$ h computer)

### **Resources:**

A classroom with a blackboard, laptop and projector for lectures and occasionally a computer room for practical sessions

# **Bibliography:**

Uriel Frisch, Turbulence, the Legacy of A. N. Kolmogorov, Cambridge University Press, 1995.

Andrew J. Majda, Andrea L. Bertozzi, Vorticity and incompressible flow, Cambridge University Press, 2002.

Philip G. Saffman, Vortex dynamics, Cambridge University Press, 1992.

Alexandre J. Chorin, Vorticity and Turbulence, Springer, 1994.

Hiroshi Kunita, Stochastic Flows and Stochastic Differential Equations, Cambridge University Press, 1990.

Peter E. Kloeden, Eckhard Platen, Numerical Solution of Stochastic Differential Equations, Springer, 1999.

Vishik, M.J., Fursikov, A.V., Mathematical Problems of Statistical Hydromechanics, Kluwer Academic Publishers, Dordrecht, 1988

## **Further comments:**